Activation Functions

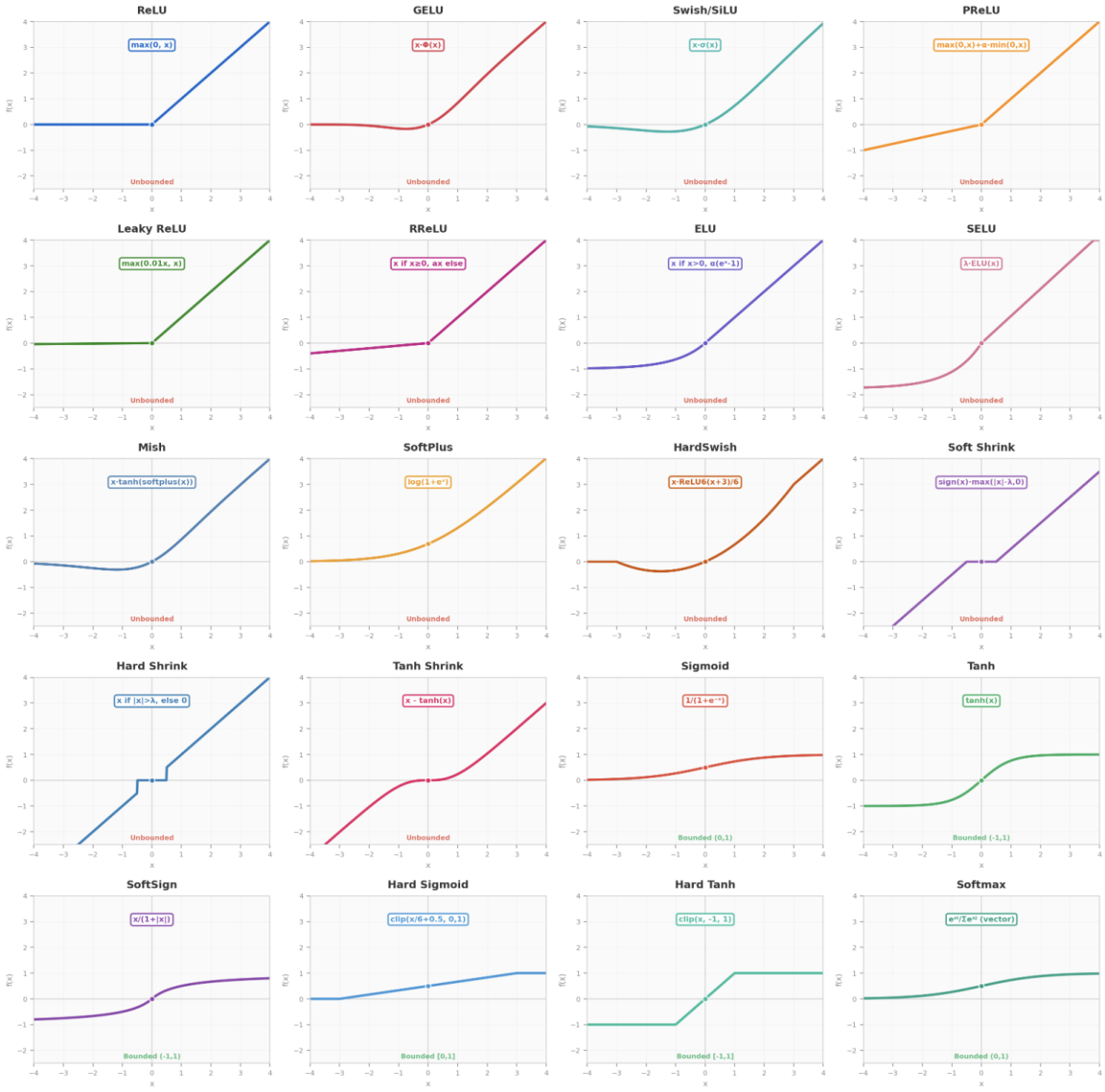

Table of Contents

The Problem Activation Functions solve

The Linearity Problem:

A neural network layer computes:

output = $W \cdot x + b$

If you stack layers without activation:

Layer 1: $h_1 = W_1 \cdot x + b_1$

Layer 2: $h_2 = W_2 \cdot h_1 + b_2 = W_2 \cdot (W_1 \cdot x + b_1) + b_2$

$= (W_2 \cdot W_1) \cdot x + (W_2 \cdot b_1 + b_2)$

$= W' \cdot x + b'$ (Still linear)

No matter how many layers it always collapses to a single linear transformation.

The Solution is non-linearity:

Layer 1: $h_1 = \text{activation}(W_1 \cdot x + b_1)$

Layer 2: $h_2 = \text{activation}(W_2 \cdot h_1 + b_2)$

Now each layer creates a new non-linear transformation that can't be simplified. This is what enables:

- Image recognition
- Language understanding
- Voice synthesis
- Game playing AI etc

Activation functions add curves to the math, enabling Deep Learning.

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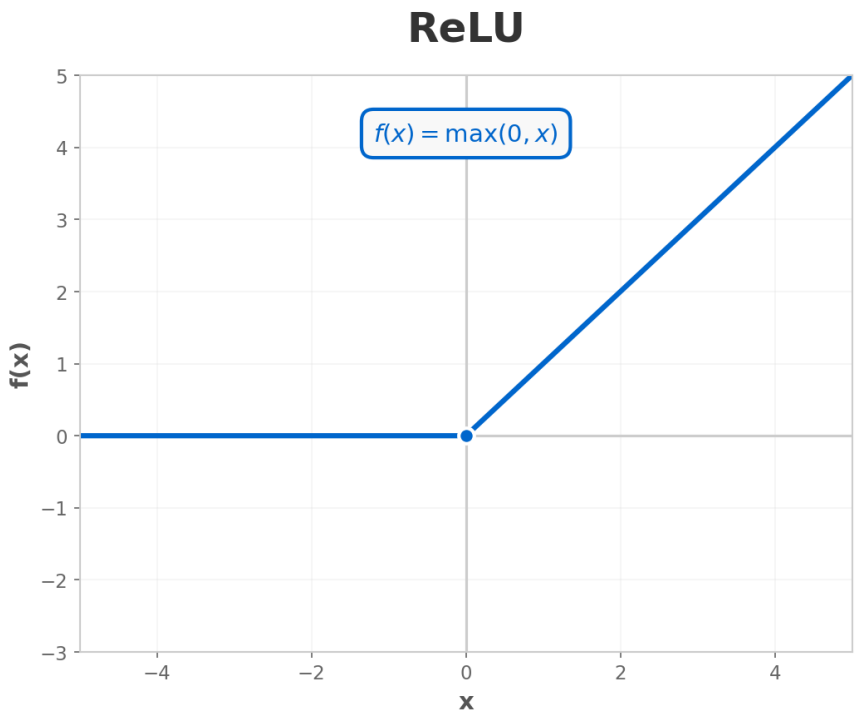
1. ReLU (Rectified Linear Unit)



Pros	Cons
Super fast (just max operation)	**Dying ReLU** - Neurons can permanently die
No vanishing gradient for positive values	Not zero-centered
Sparse activation (efficient)	Unbounded (can explode)

Why it's popular: Simple, fast, works well in most CNNs.

Used in: AlexNet, VGG, ResNet, most CNN architectures

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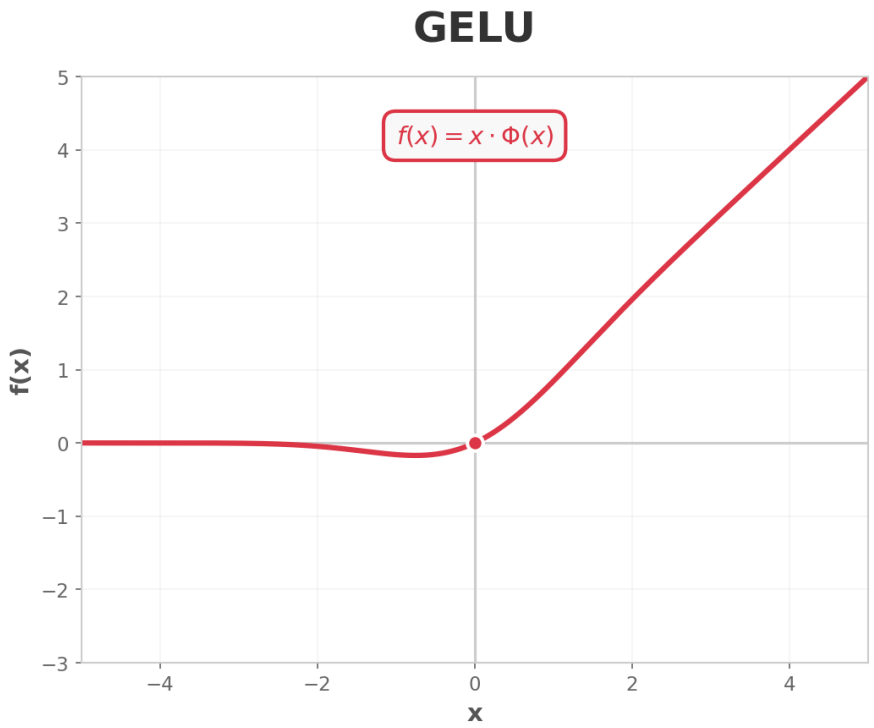
2. GELU (Gaussian Error Linear Unit)



Pros	Cons
Smooth, differentiable everywhere	Computationally expensive
Probabilistic interpretation	Slower than ReLU
No dying Neuron problem	
Better gradient flow	

Why it's better than ReLU: Smoothness helps transformers learn better. Small negative values aren't completely killed.

Used in: GPT-2, GPT-3, GPT-4, BERT, Vision Transformer (ViT)

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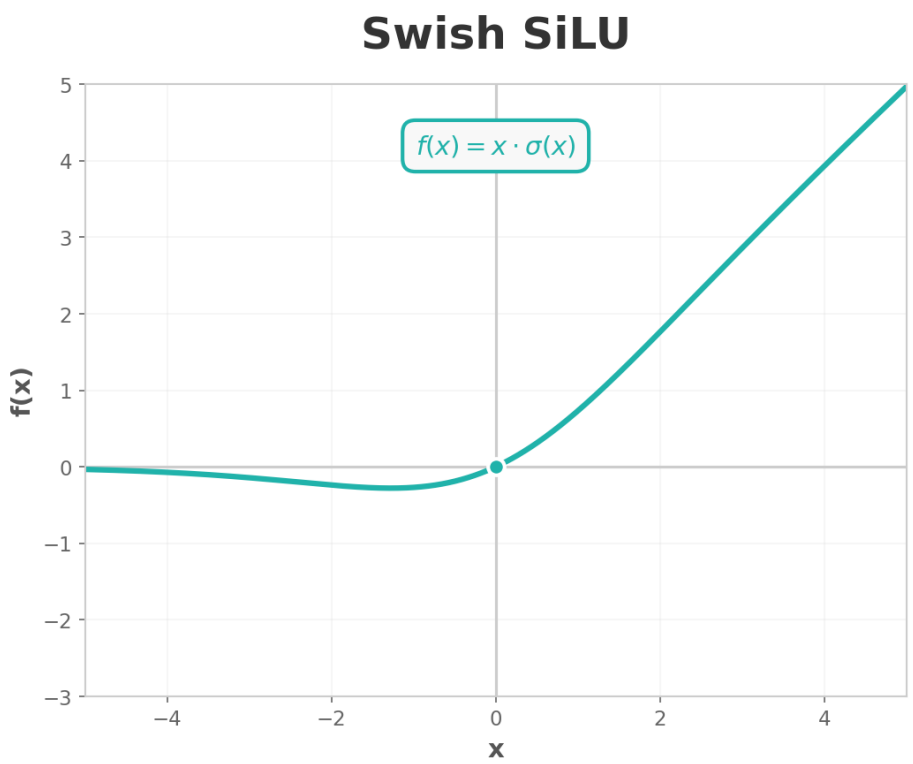
3. Swish / SiLU (Sigmoid Linear Unit)



Pros	Cons
Smooth & non-monotonic	More compute than ReLU
Self-gating property	Unbounded above
Better gradient flow	
Outperforms ReLU in deep networks	

Why it's better than ReLU: The smooth curve and self-gating allow negative values to contribute slightly, preventing dead neurons.

Used in: EfficientNet, LLaMA 1-4, Mistral, Qwen, MobileNetV3 etc

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4. SwiGLU (Swish-Gated Linear Unit)

Pros	Cons
State-of-the-art for LLMs	50% more parameters
Gating mechanism adds expressiveness	More memory usage
Better gradient flow	Can't visualize as 2D curve
Outperforms GELU in transformers	

Why it's the 2024-25 king: Combines the smoothness of Swish with gating mechanism. Empirically dominates benchmarks (researchers say it **divine benevolence**).

Used in: LLaMA 2-4, PaLM, Mistral, Mixtral, Qwen 3, DeepSeek-V3

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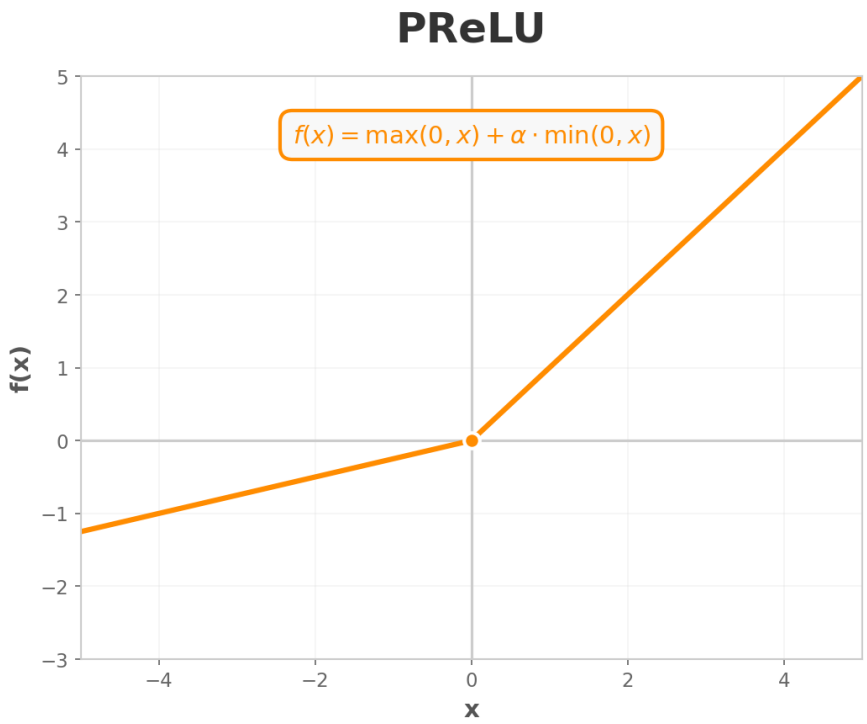
5. PReLU (Parametric ReLU)



Pros	Cons
Fixes dying ReLU	Extra parameter to learn
Learnable slope	Can overfit on small datasets
Network decides best α	

Why it's better than ReLU: The network learns the optimal slope for negatives instead of using fixed 0.

Used in: ResNets, face recognition models

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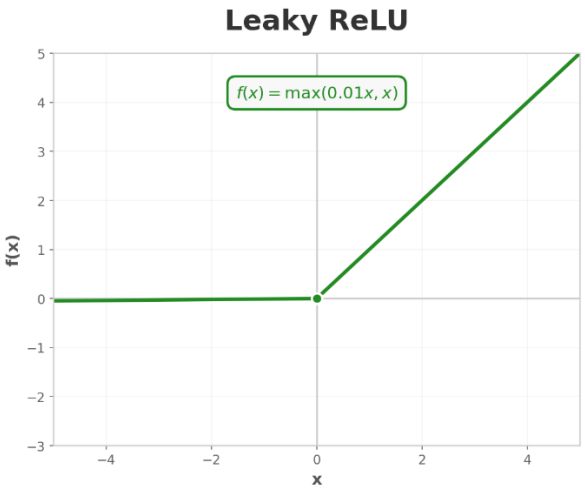
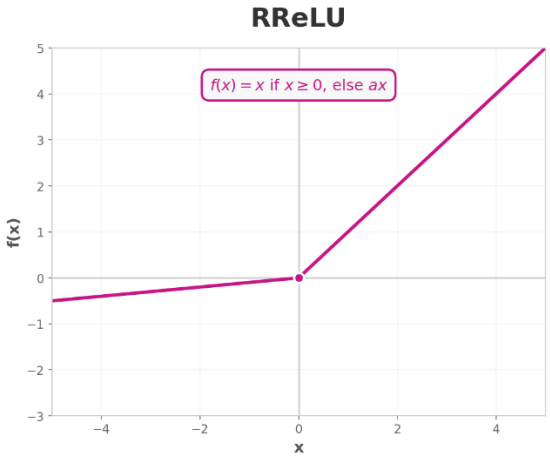
6. Leaky ReLU / RReLU





Pros	Cons
Prevents dying ReLU	Fixed α may not be optimal
Simple fix to ReLU	Still not smooth
RReLU adds regularization	

Why it's better than ReLU: Small gradient for negatives keeps neurons alive.

Used in: YOLO variants, GANs

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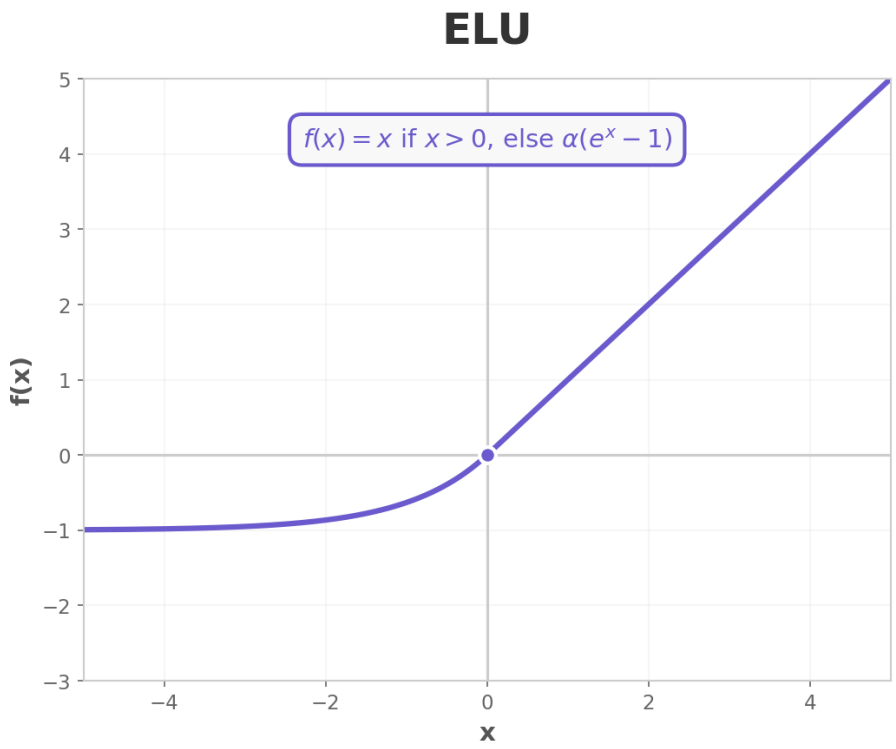
7. ELU (Exponential Linear Unit)



Pros	Cons
Smooth for negatives	Exponential is slow
Outputs closer to zero-mean	Saturates for very negative values
Faster convergence than ReLU	

Why it's better than ReLU: The smooth negative part helps with gradient flow and produces zero-centered outputs.

Used in: Deep networks where batch normalization isn't used

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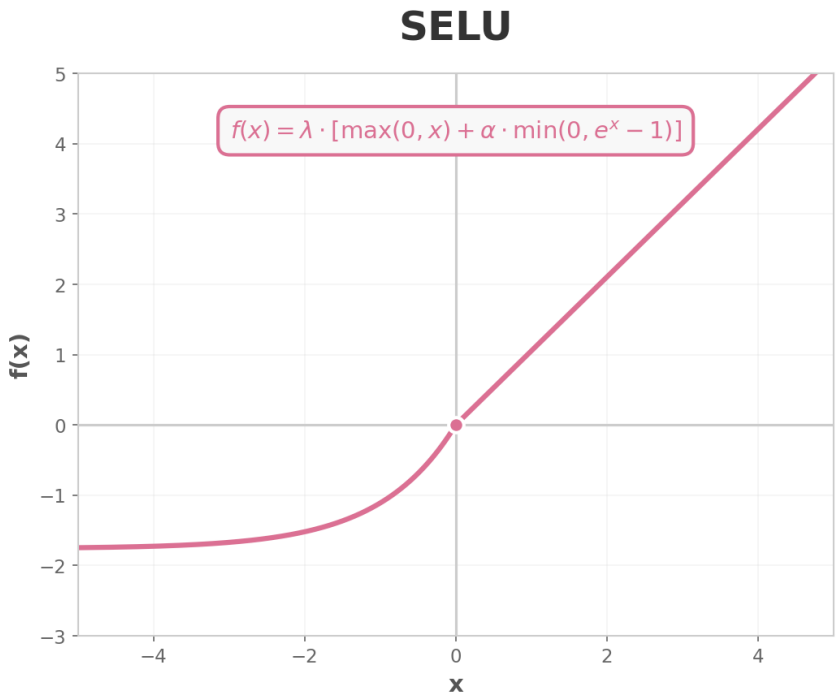
8. SELU (Scaled ELU)



Pros	Cons
Self-normalizing	Only works with specific initialization
No need for batch norm	Requires LeCun normal init
Great for deep MLPs	Not ideal for CNNs/RNNs

Why it's special: Mathematically designed to maintain mean=0, variance=1 through layers. Built-in normalization.

Used in: Very deep fully-connected networks

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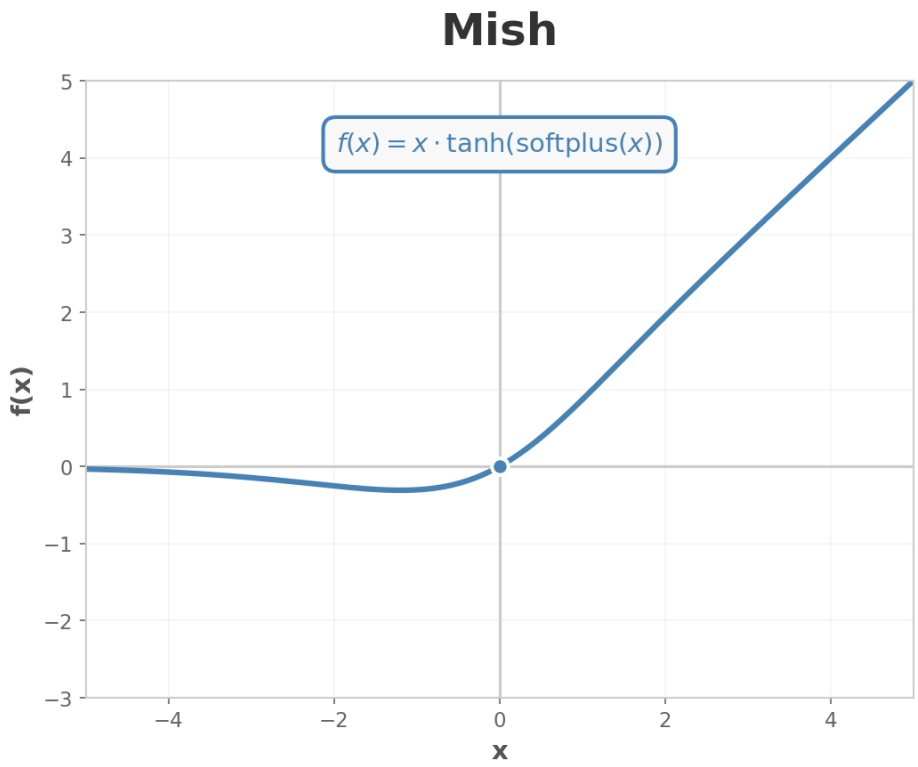
9. Mish



Pros	Cons
Smoother than Swish	Computationally expensive
Unbounded above, bounded below	Complex formula
Self-regularizing	
No dying neurons	

Why it's better than ReLU/Swish: Extra smoothness from tanh(softplus) provides even better gradient flow in deep networks.

Used in: YOLOv4, YOLOv5, object detection models

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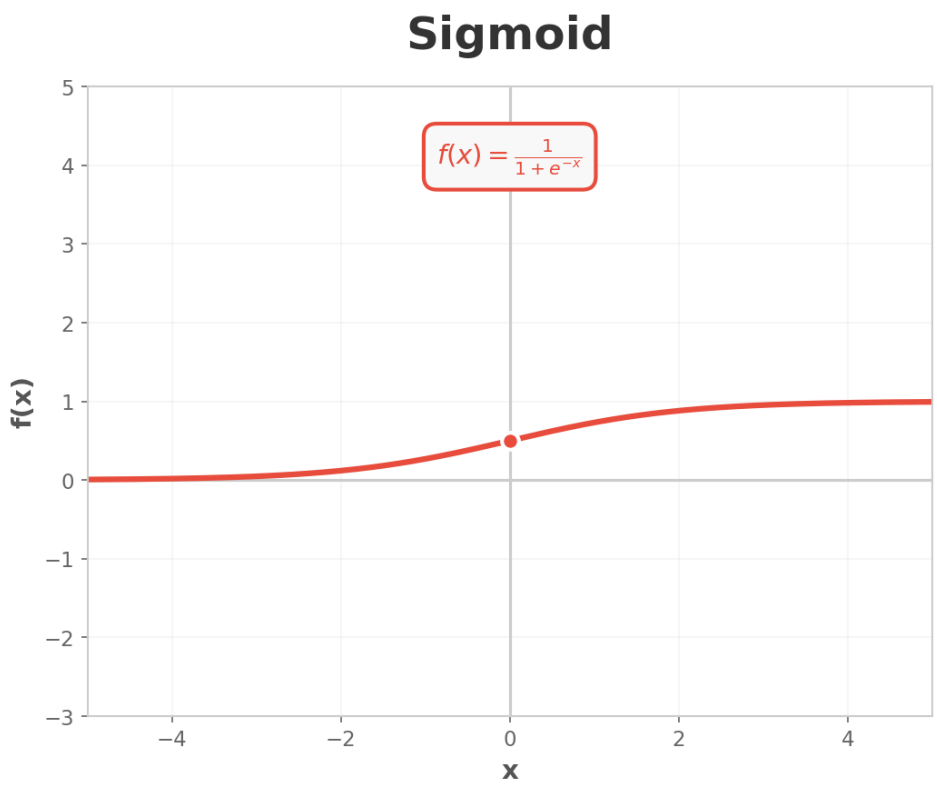
10. Sigmoid



Pros	Cons
Output bounded (0, 1)	Vanishing gradient problem
Probabilistic interpretation	Not zero-centered
Good for output layers	Saturates at extremes

Why it's NOT used in hidden layers anymore: Gradients vanish when x is very positive or negative → deep networks can't learn.

Still used for: Binary classification output, gates in LSTMs

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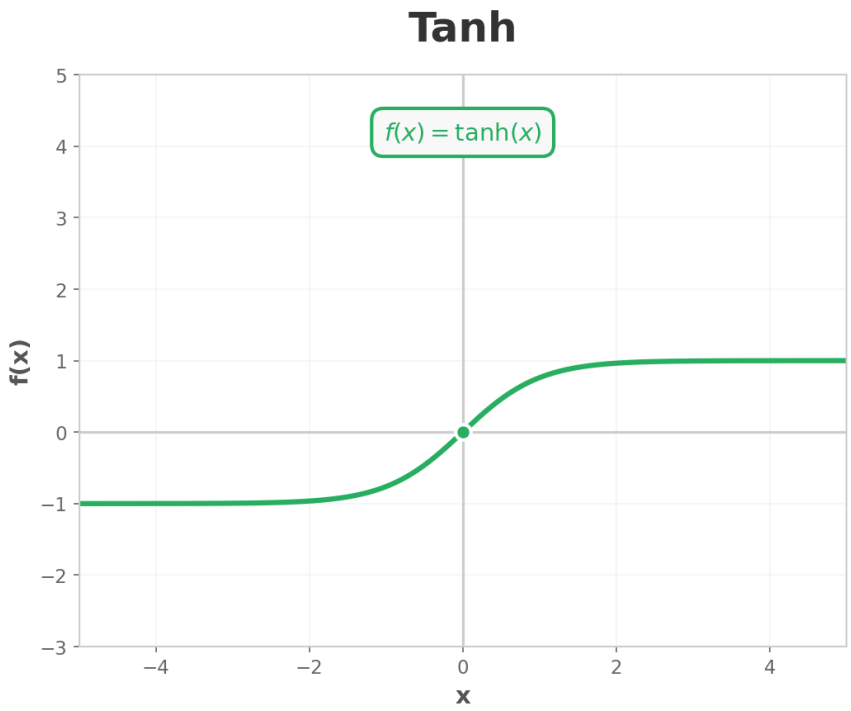
11. Tanh



Pros	Cons
Zero-centered (-1 to 1)	Vanishing gradient
Stronger gradients than sigmoid	Saturates at extremes
Good for normalized data	

Why it's better than Sigmoid: Zero-centered outputs help with convergence. But still has vanishing gradient.

Used in: RNNs, LSTMs, GRUs (for gates and states)

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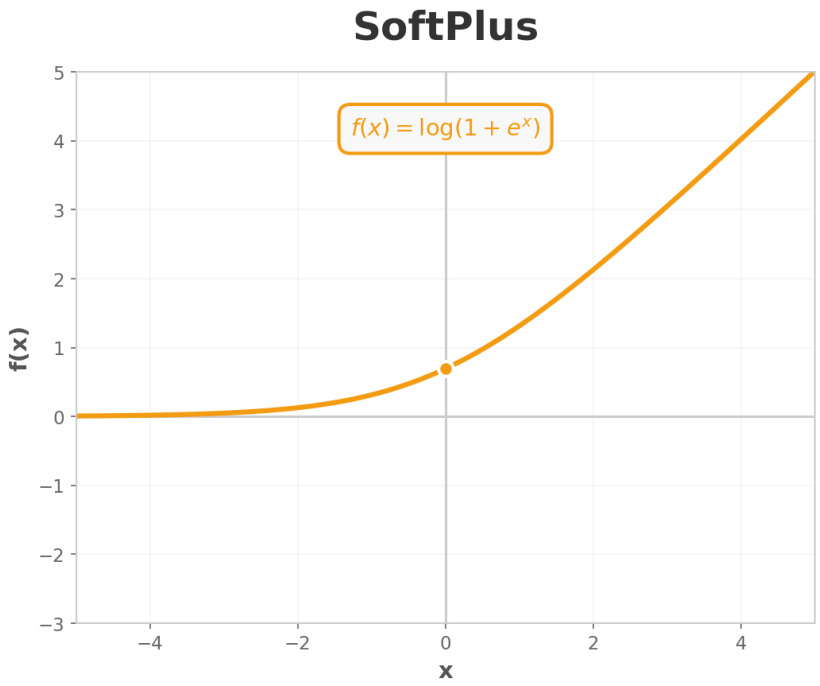
12. SoftPlus



Pros	Cons
Smooth version of ReLU	Never exactly zero
Always positive output	Slower than ReLU
Differentiable everywhere	

Why it exists: Smooth approximation to ReLU when you need continuous derivatives.

Used in: Probabilistic models, variational autoencoders

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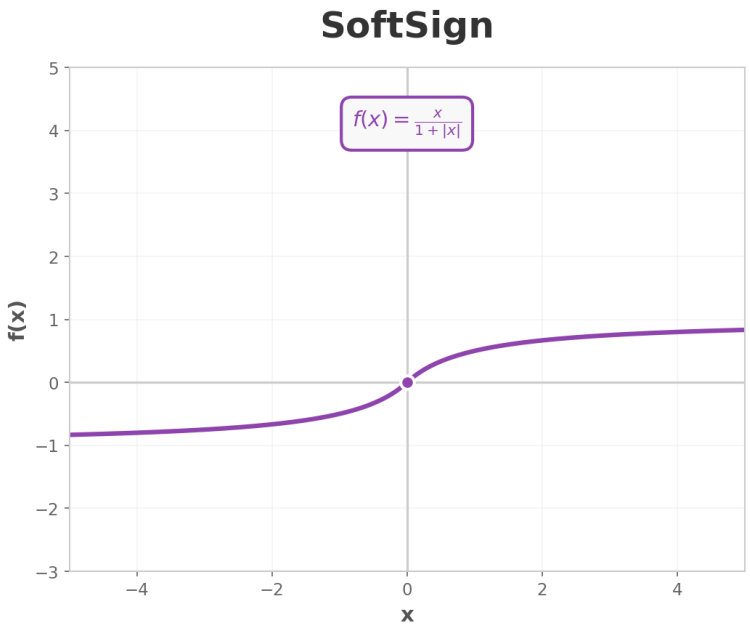
13. SoftSign



Pros	Cons
Smoother than Tanh	Slower convergence
Doesn't saturate as fast	Less commonly used
Zero-centered	

Why it's better than Tanh: Approaches ±1 more slowly → less gradient vanishing.

Used in: Some NLP applications, when Tanh saturates too fast

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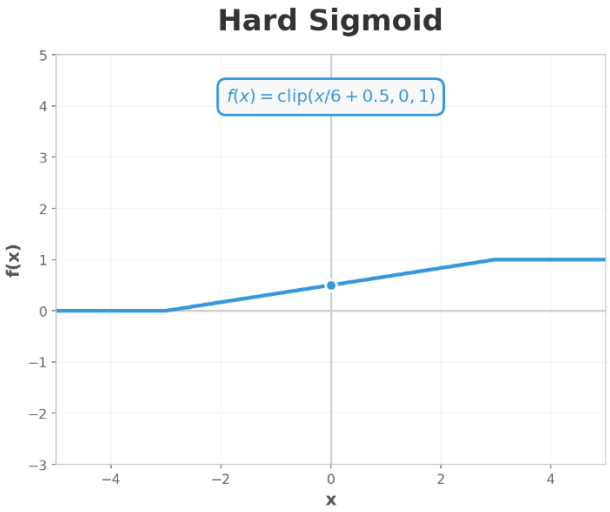
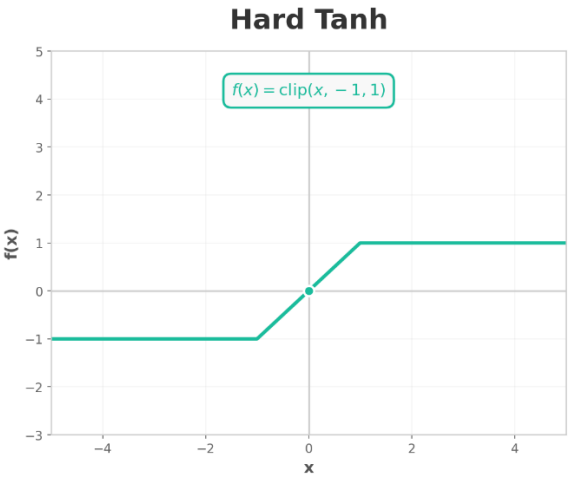
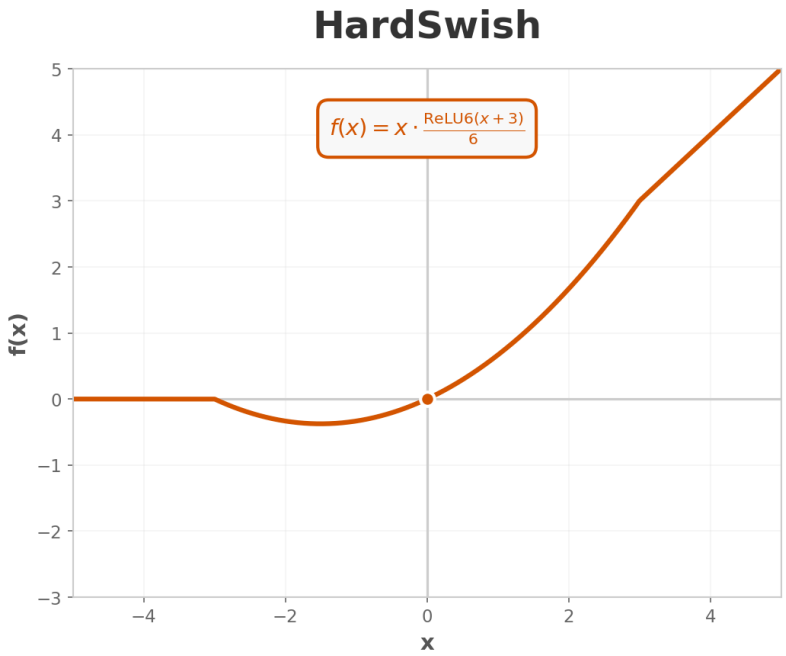
14. Hard Sigmoid / Hard Tanh / HardSwish







Pros	Cons
Computationally cheap	Not smooth (has kinks)
No exponentials	Less accurate approximation
Great for mobile/edge	

Why they exist: Fast approximations for resource-constrained devices. Trade smoothness for speed.

Used in: MobileNetV3, edge AI, quantized models

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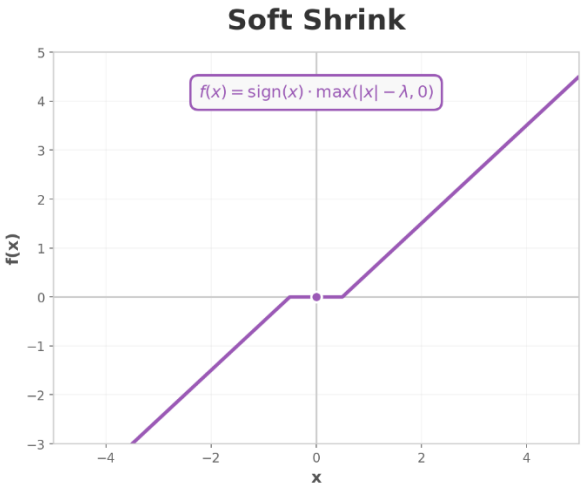
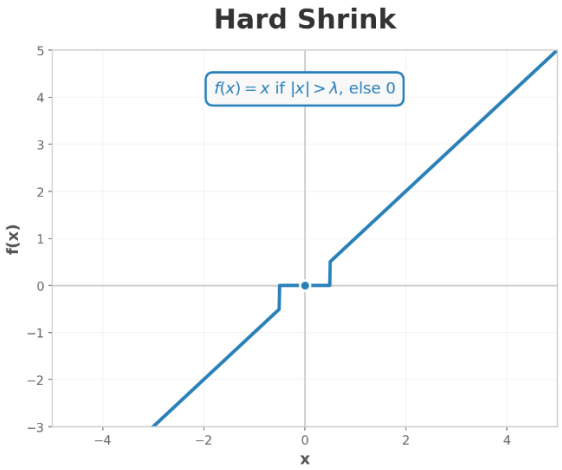
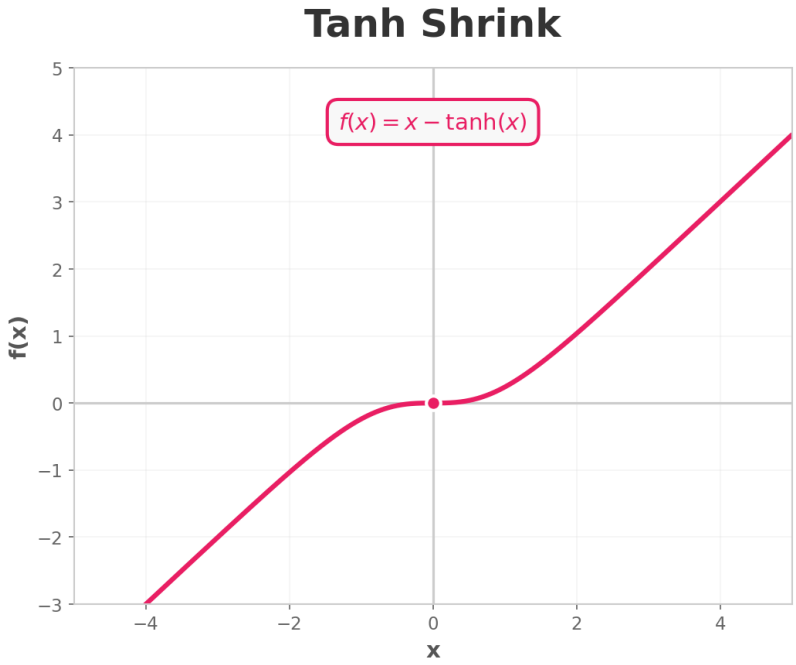
15. Shrink Functions (Soft/Hard/Tanh Shrink)







Pros	Cons
Sparsity-inducing	Niche applications
Good for denoising	Not mainstream
Thresholding behaviour	

Why they exist: Push small values to zero → sparse representations, useful in autoencoders and signal processing.

Used in: Wavelet transforms, sparse coding, denoising autoencoders

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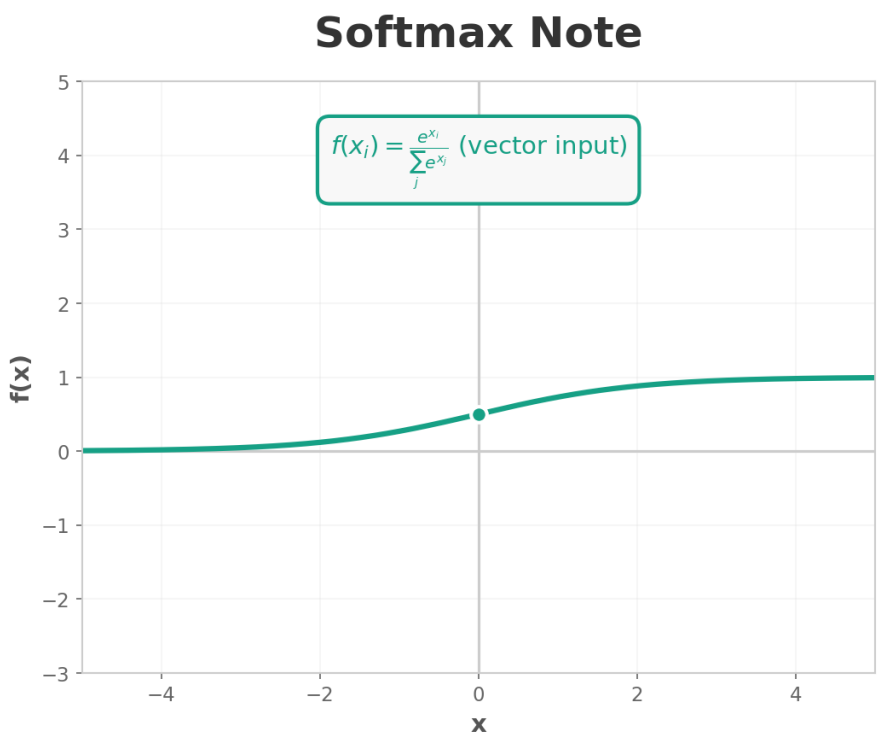
16. Softmax



Pros	Cons
Outputs sum to 1	Only for output layer
Probabilistic interpretation	Computationally expensive
Works on vectors	Can't use in hidden layers

Why it's different: Operates on entire vector, not single values. Turns scores into probabilities.

Used in: Classification output layer, LLM next-token prediction (GPT-4, LLaMA, Claude etc)

Ranking: Best for each Use Case

Use Case	Best Choice	Why
CNN hidden layers	ReLU, Mish	Fast, works well
Transformer/LLM FFN	SwiGLU, GELU	Smooth gradients, better performance
RNN/LSTM gates	Sigmoid, Tanh	Bounded outputs needed
Very deep MLPs	SELU	Self-normalizing
Mobile/Edge AI	HardSwish, ReLU	Computationally efficient
Binary classification output	Sigmoid	Outputs probability
Multi-class output	Softmax	Probabilities sum to 1
Regression output	Linear (None)	Unbounded output needed
Object detection	Mish, Leaky ReLU	Better gradient flow
GANs	Leaky ReLU	Prevents dying neurons

Evolution Timeline

1990s: Sigmoid, Tanh (vanishing gradient problems)

↓

2010: ReLU (solved vanishing gradient, but dying neurons)

↓

2015: Leaky ReLU, PReLU, ELU (fixed dying neurons)

↓

2016: SELU (self-normalizing)

↓

2017: Swish/SiLU (smooth, self-gating)

↓

2018: GELU (transformers love it)

↓

2019: Mish (even smoother)

↓

2020: SwiGLU, GeGLU (gated FFN layers)

↓

2024-25: SwiGLU dominates LLMs

Key takeaways

1. **ReLU** is still the default starting point, simple and fast
2. **GELU/SwiGLU** for transformers and LLMs, smooth gradients matter
3. **Sigmoid/Tanh** only for output layers or RNN gates now
4. **Hard variants** for mobile/edge deployment
5. **No single best** - depends on architecture, task and compute budget